International Journal of Innovative Research in Engineering & Management (IJIREM)
ISSN: 2350-0557, Volume-3, Issue-4, July-2016

Solving Multi-Level Programming Problem with Interval Coefficients by Using Fuzzy Programming

Samir A. Abass
Atomic Energy Authority, Nuclear Research Center, Cairo, Egypt.
P.O. Box 13759, Egypt.

Asmaa S. Abdallah
Atomic Energy Authority, Nuclear Research Center, Cairo, Egypt.
P.O. Box 13759, Egypt.

ABSTRACT

This paper proposes a solution method for three level programming (TLP) problem with interval coefficients in both of objective functions and constraints. This method uses the concepts of tolerance membership function at each level to develop a fuzzy Max–Min decision model for generating Pareto optimal (satisfactory) solution. The first level decision maker (FLDM) specifies his/her objective functions and decisions with possible tolerances. Then, the second level decision-maker (SLDM) specifies his/her objective functions and decisions, in the view of the FLDM, with possible. Finally, the third level decision-maker (TLDM) uses the preference information for the FLDM and SLDM to solve his/her problem subject to the two upper level decision-makers restrictions. An illustrative numerical example is provided to clarify the proposed approach.

Keywords: Multi-level programming, interval coefficients, fuzzy programming.

1. INTRODUCTION

Three-level programming (TLP) problem is a class of multi-level programming (MLP) problem in which there are three independent decision makers [3, 5, 14, 16]. Each DM attempts to optimize its objective function and is affected by the actions of the others decision makers. TLP problem, whether from the stand point of the three-planner Stackelberg behavior or from the interactive organizational behavior, is a very practical problem and encountered frequently in actual practice.

Several approaches have been used widely in searching for the optimal solutions of three level programming problems. The hybrid extreme-point search algorithm [4, 7], mixed-integer problem with complementary slackness [7], and the penalty function approach [2, 7]. Apart from these approaches, fuzzy sets have been employed to formulate and solve three-level non-linear multi-objective decision-making [12]. A multi-level linear programming problem with random rough coefficients in objective functions was presented by Emam et al [6].

In traditional mathematical programming, the coefficients of the problems are always treated as deterministic values. However in practice, it is very common for the coefficients values to be only approximately known. Hence, in order to develop good Operations Research methodology, fuzzy and stochastic approaches are frequently used to describe and treat uncertain elements present in a real decision problem. In fuzzy programming, the constraints and goals are viewed as fuzzy sets and it is assumed that their membership functions are known. On the other hand, in stochastic programming, the coefficients are viewed as random variables and it is also assumed that their probability distributions are known. These membership functions and probability distributions play important roles in their corresponding methods. However, it is sometimes difficult to specify an appropriate membership function or accurate probability distribution in an uncertain environment. To overcome these difficulties, interval programming approaches [9] have appeared as a prominent tool for solving decision problems with interval parameter values. In interval programming the bounds of the uncertain coefficients are only required. The methodological aspects of interval programming studied in the past have been surveyed by Olivera et al [11].

The structure of this paper is organized as follows. Section 2 presents the TLP problem with interval coefficients in both of the objective functions and constraints. Section 3 presents treatment procedure of the uncertainty of the problem. Fuzzy models for TLP problem are provided in section 4. A numerical example is provided in section 5 to clarify the proposed approach. Section 6 contains the conclusion.

2. THREE-LEVEL PROGRAMMING PROBLEM WITH INTERVAL COEFFICIENTS

The general form of TLP problem with interval coefficients in both of the objective functions and the constraints is as follows:

$$\max_{\bar{x}_1} f_1(\bar{x}) = \left[c_{11}^L, c_{11}^R\right]\bar{x}_1 + \left[c_{12}^L, c_{12}^R\right]\bar{x}_2 + \left[c_{13}^L, c_{13}^R\right]\bar{x}_3, \tag{1}$$

where $\bar{x}_2, \bar{x}_3$ solve:

$$\max_{\bar{x}_2} f_2(\bar{x}) = \left[c_{21}^L, c_{21}^R\right]\bar{x}_1 + \left[c_{22}^L, c_{22}^R\right]\bar{x}_2 + \left[c_{23}^L, c_{23}^R\right]\bar{x}_3, \tag{2}$$

and $\bar{x}_3$ solves:

$$\max_{\bar{x}_3} f_3(\bar{x}) = \left[c_{31}^L, c_{31}^R\right]\bar{x}_1 + \left[c_{32}^L, c_{32}^R\right]\bar{x}_2 + \left[c_{33}^L, c_{33}^R\right]\bar{x}_3, \tag{3}$$

subject to

$$\left[a_{i1}^L, a_{i1}^R\right]\bar{x}_1 + \left[a_{i2}^L, a_{i2}^R\right]\bar{x}_2 + \left[a_{i3}^L, a_{i3}^R\right]\bar{x}_3 \le b_i \quad, i = 1, 2, ..., m$$

(4)

$$\overline{x}_1 \geq 0,\ \overline{x}_2 \geq 0,\ \overline{x}_3 \geq 0 \qquad (5)$$

where $\overline{x}_1 = \{x_1^1, x_1^2, ..., x_1^{n_1}\}$ and n_1 is the number of variables controlled by the first level. $\overline{x}_2 = \{x_2^1, x_2^2, ..., x_2^{n_2}\}$ and n_2 is the number of variables controlled by the second level. $\overline{x}_3 = \{x_3^1, x_3^2, ..., x_3^{n_2}\}$ and n_3 is the number of variables controlled by the third level. $\overline{x} = \{\overline{x}_1, \overline{x}_2, \overline{x}_3\}$ is the decision variables controlled by the decision maker on the three levels. $n = n_1 + n_2 + n_3$ and m is the number of constraints. $[a_i^L, a_i^R]$, for $i = 1,...,m$ and $[c^L, c^R]$ denote an interval numbers and they represent a bounded set of real numbers between the bounds. The superscripts L and R denote lower and upper bounds of an interval number respectively.

3. TREATMENT PROCEDUOR

In this section we will treat the uncertainty in both of the objective functions and the constraints.

3.1 Treatment of the uncertain objective function

In interval mathematics, the uncertain objective function with interval numbers can be transformed into two deterministic objective functions as follows (Jiang et al., 2008):

$$m\left(f_1\left(\overline{x}\right)\right) = \frac{1}{2}\left(f_1^L\left(\overline{x}\right) + f_1^R\left(\overline{x}\right)\right),$$

$$w\left(f_1\left(\overline{x}\right)\right) = \frac{1}{2}\left(f_1^R\left(\overline{x}\right) - f_1^L\left(\overline{x}\right)\right),$$

where m is called the midpoint value and w is called the radius of the interval number.

The two functions $f_1^L\left(\overline{x}\right)$ and $f_1^R\left(\overline{x}\right)$ are given as follows:

$$f_1^L\left(\overline{x}\right) = \min f_1\left(\overline{x}\right),\quad f_1^R\left(\overline{x}\right) = \max f_1\left(\overline{x}\right)$$

The linear combination method [8] is adopted to deal with the multi objective optimization. The linear combination method guarantees obtainment finite number of Pareto solutions. So it is appropriate for this type of problems, then we get:

$$f_1\left(\overline{x}\right) = d_1 m\left(f_1\left(\overline{x}\right)\right) + d_2 w\left(f_1\left(\overline{x}\right)\right),$$

$$d_1, d_2 \geq 0,\quad d_1 + d_2 = 1$$

And by the same way the objective functions of the second and third levels can be written as follows:

$$f_2\left(\overline{x}\right) = d_3 m\left(f_2\left(\overline{x}\right)\right) + d_4 w\left(f_2\left(\overline{x}\right)\right)$$

$$d_3, d_4 \geq 0,\quad d_3 + d_4 = 1$$

$$f_3\left(\overline{x}\right) = d_5 m\left(f_3\left(\overline{x}\right)\right) + d_6 w\left(f_3\left(\overline{x}\right)\right)$$

$$d_5, d_6 \geq 0,\quad d_5 + d_6 = 1$$

where d_1, d_2, d_3, d_4, d_5 and d_6 are called weighting coefficients.

3.2 Treatment of the uncertain constraints

The possibility degree of interval number represents certain degree that one interval number is larger or smaller than another.

Let $g_i\left(\overline{x}\right) = \left[a_{i1}^L, a_{i1}^R\right]\overline{x}_1 + \left[a_{i2}^L, a_{i2}^R\right]\overline{x}_2 + \left[a_{i3}^L, a_{i3}^R\right]\overline{x}_3$

According to Jiang et al. [10], the possibility degree can be defined as follows:

$$P_{(b \geq E)} = \begin{cases} 0 & b < E^L \\ \dfrac{b - E^L}{E^R - E^L} & E^L \leq b < E^R \\ 1 & b \geq E^R \end{cases}$$

where $E = \left[g_i^L\left(\overline{x}\right), g_i^R\left(\overline{x}\right)\right]$ is the interval of the ith constraint function.

$P_{(b \geq E)} \geq \lambda_i$ is the possibility degree of the ith constraint where $0 \leq \lambda_i \leq 1$, $i = 1, 2, ..., m$ is a predetermined possibility degree level.

By applying the possibility degree definition, we can transform the uncertain form of constraints (4) into certain form as follows:

$$P_{\left(b_i \geq \left[a_{i1}^L, a_{i1}^R\right]\overline{x}_1 + \left[a_{i2}^L, a_{i2}^R\right]\overline{x}_2 + \left[a_{i3}^L, a_{i3}^R\right]\overline{x}_3\right)} \geq \lambda_i \quad ,$$

$$i = 1, 2, ..., m$$

then

$$\frac{b_i - \left(a_{i1}^L\overline{x}_1 + a_{i2}^L\overline{x}_2 + a_{i3}^L\overline{x}_3\right)}{\left(a_{i1}^R\overline{x}_1 + a_{i2}^R\overline{x}_2 + a_{i3}^R\overline{x}_3\right) - \left(a_{i1}^L\overline{x}_1 + a_{i2}^L\overline{x}_2 + a_{i3}^L\overline{x}_3\right)} \geq \lambda_i \quad ,$$

$$i = 1, 2, ..., m$$

$$0 \leq \lambda_i \leq 1$$

3.3 The deterministic TLP problem

Now we have a deterministic form of TLP problem as follows

$$\max_{x_1} f_1\left(\overline{x}\right) = d_1 m\left(f_1\left(\overline{x}\right)\right) + d_2 w\left(f_1\left(\overline{x}\right)\right) \qquad (6)$$

where $\overline{x}_2, \overline{x}_3$ solve:

$$\max_{x_2} f_2\left(\overline{x}\right) = d_3 m\left(f_2\left(\overline{x}\right)\right) + d_4 w\left(f_2\left(\overline{x}\right)\right) \qquad (7)$$

and $\overline{x}_3$ solves:

$$\max_{x_3} f_3\left(\overline{x}\right) = d_5 m\left(f_3\left(\overline{x}\right)\right) + d_6 w\left(f_3\left(\overline{x}\right)\right)$$

(8)

subject to

$$\frac{b_i-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}{\left(a_{i1}^R\overline{x}_1+a_{i2}^R\overline{x}_2+a_{i3}^R\overline{x}_3\right)-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}\ge\lambda_i\ ,\quad i=1,2,\dots,m \tag{9}$$

$$0\le\lambda_i\le 1 \tag{10}$$

$$d_1,d_2,d_3,d_4,d_5,d_6\ge 0,\quad d_1+d_2=1,\ d_3+d_4=1,\ d_5+d_6=1 \tag{11}$$

$$\overline{x}_1\ge 0,\ \overline{x}_2\ge 0,\ \overline{x}_3\ge 0 \tag{12}$$

For some values of $\lambda_i, d_1, d_2, d_3, d_4, d_5$ and d_6 we can treat problem (6) – (12) by using fuzzy programming.

4. FUZZY MODELS FOR TLP PROBLEM

To solve the TLP problem one first gets the satisfactory solution that is acceptable to FLDM, and then gives decision variables and goals of the FLDM with some leeway to the SLDM for him/her to seek the satisfactory solution. Then give the decision variables and goals of the SLDM with some leeway to the TLDM for him/her to seek the satisfactory solution, and to arrive at the solution which is closest to the satisfactory solution of the FLDM. This due to, the TLDM who should not only optimize his/her objective functions but also try to satisfy the goals and preferences of SLDM as much as possible, SLDM also do the same action to satisfy the goals and preferences of FLDM as much as possible.

The TLP problem will be solved as a single objective problem considering one objective at a time and ignoring the rest.

By using the obtained solutions, we find the values of all objective functions at each solution and construct a payoff matrix as follows [15]:

$$\begin{array}{c} \\ \overline{x}_1^o \\ \overline{x}_2^o \\ \overline{x}_3^o \end{array}\begin{array}{c} \begin{array}{ccc} f_1(\overline{x}) & f_2(\overline{x}) & f_3(\overline{x}) \end{array} \\ \begin{bmatrix} f_1(\overline{x}_1^o) & f_2(\overline{x}_1^o) & f_3(\overline{x}_1^o) \\ f_1(\overline{x}_2^o) & f_2(\overline{x}_2^o) & f_3(\overline{x}_2^o) \\ f_1(\overline{x}_3^o) & f_2(\overline{x}_3^o) & f_3(\overline{x}_3^o) \end{bmatrix} \end{array}$$

The minimum and maximum of each column gives us lower bound and upper bound of each objective function then we have:

$$f_1^u(\overline{x})=\max f_1(\overline{x})\ \text{and}\ f_1^l(\overline{x})=\min f_1(\overline{x}), \tag{13}$$

$$f_2^u(\overline{x})=\max f_2(\overline{x})\ \text{and}\ f_2^l(\overline{x})=\min f_2(\overline{x}), \tag{14}$$

$$f_3^u(\overline{x})=\max f_3(\overline{x})\ \text{and}\ f_3^l(\overline{x})=\min f_3(\overline{x}) \tag{15}$$

Equations (13), (14) and (15) will be used to build the membership functions of fuzzy set theory [13].

4.1. FLDM problem

By using (13) the following the membership functions of fuzzy set theory can be formulated:

$$\mu_{f_1}\left[f_1(\overline{x})\right]=\begin{cases}0 & f_1(\overline{x})\le f_1^l(\overline{x}),\\ \dfrac{f_1(\overline{x})-f_1^l(\overline{x})}{f_1^u(\overline{x})-f_1^l(\overline{x})} & f_1^l(\overline{x})\le f_1(\overline{x})\le f_1^u(\overline{x}),\\ 1 & f_1(\overline{x})\ge f_1^u(\overline{x}).\end{cases} \tag{16}$$

Now, we can get the solution of the FLDM problem by solving the following Tchebycheff problem [1, 7, 14]:

$$\max\ \alpha \tag{17}$$

subject to:

$$\mu_{f_1}\left[f_1(\overline{x})\right]\ge\alpha \tag{18}$$

$$\frac{b_i-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}{\left(a_{i1}^R\overline{x}_1+a_{i2}^R\overline{x}_2+a_{i3}^R\overline{x}_3\right)-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}\ge\lambda_i\ ,\quad i=1,2,\dots,m \tag{19}$$

$$0\le\lambda_i\le 1 \tag{20}$$

$$d_1,d_2\ge 0,\quad d_1+d_2=1 \tag{21}$$

$$\overline{x}_1\ge 0,\ \overline{x}_2\ge 0,\ \overline{x}_3\ge 0 \tag{22}$$

Whose solutions are assumed to be $\left[\overline{x}_1^F,\overline{x}_2^F,\overline{x}_3^F,f_1^F,\alpha^F\ (\text{satisfactory level})\right]$.

4.2. SLDM problem

By using (14) the following the membership functions can be formulated:

$$\mu_{f_2}\left[f_2(\overline{x})\right]=\begin{cases}0 & f_2(\overline{x})\le f_2^l(\overline{x}),\\ \dfrac{f_2(\overline{x})-f_2^l(\overline{x})}{f_2^u(\overline{x})-f_2^l(\overline{x})} & f_2^l(\overline{x})\le f_2(\overline{x})\le f_2^u(\overline{x}),\\ 1 & f_2(\overline{x})\ge f_2^u(\overline{x}).\end{cases} \tag{23}$$

Now, we can get the solution of the SLDM problem by solving the following Tchebycheff problem:

$$\max\beta \tag{24}$$

subject to:

$$\mu_{f_2}\left[f_2(\overline{x})\right]\ge\beta \tag{25}$$

$$\frac{b_i-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}{\left(a_{i1}^R\overline{x}_1+a_{i2}^R\overline{x}_2+a_{i3}^R\overline{x}_3\right)-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}\ge\lambda_i\ ,\quad i=1,2,\dots,m \tag{26}$$

$$0\le\lambda_i\le 1 \tag{27}$$

$$d_3,d_4\ge 0,\quad d_3+d_4=1 \tag{28}$$

$$\overline{x}_1\ge 0,\ \overline{x}_2\ge 0,\ \overline{x}_3\ge 0 \tag{29}$$

Whose solutions are assumed to be:

$$\left[\bar{x}_1^S, \bar{x}_2^S, \bar{x}_3^S, f_2^S, \beta^S \text{ (satisfactory level)}\right].$$

4.3. TLDM problem

By using (15) the following the membership functions can be formulated:

$$\mu_{f_3}\left[f_3(\bar{x})\right] = \begin{cases} 0 & f_3(\bar{x}) \le f_3^l(\bar{x}), \\ \dfrac{f_3(\bar{x}) - f_3^l(\bar{x})}{f_3^u(\bar{x}) - f_3^l(\bar{x})} & f_3^l(\bar{x}) \le f_3(\bar{x}) \le f_3^u(\bar{x}), \\ 1 & f_3(\bar{x}) \ge f_3^u(\bar{x}). \end{cases} \quad (30)$$

Now, we can get the solution of the TLDM problem by solving the following Tchebycheff problem:

$$\max \gamma \quad (31)$$

subject to:

$$\mu_{f_3}\left[f_3(\bar{x})\right] \ge \gamma \quad (32)$$

$$\frac{b_i - \left(a_{i1}^L \bar{x}_1 + a_{i2}^L \bar{x}_2 + a_{i3}^L \bar{x}_3\right)}{\left(a_{i1}^R \bar{x}_1 + a_{i2}^R \bar{x}_2 + a_{i3}^R \bar{x}_3\right) - \left(a_{i1}^L \bar{x}_1 + a_{i2}^L \bar{x}_2 + a_{i3}^L \bar{x}_3\right)} \ge \lambda_i, \quad i = 1, 2, \ldots, m \quad (33)$$

$$0 \le \lambda_i \le 1 \quad (34)$$

$$d_5, d_6 \ge 0, \quad d_5 + d_6 = 1 \quad (35)$$

$$\bar{x}_1 \ge 0,\ \bar{x}_2 \ge 0,\ \bar{x}_3 \ge 0 \quad (36)$$

Whose solutions are assumed to be:

$$\left[\bar{x}_1^T, \bar{x}_2^T, \bar{x}_3^T, f_3^T, \gamma^T \text{ (satisfactory level)}\right].$$

4.4 TLP problem

Now the solution of the FLDM, SLDM, and TLDM are disclosed. However, three solutions are usually different because of nature between three levels objective functions. The FLDM and SLDM knows that using the optimal decisions $\bar{x}_1^F, \bar{x}_2^S$ as a control variables for the TLDM are not practical. It is more reasonable to have some tolerance that gives the TLDM an extent feasible region to search for his/her optimal solution, and also reduce searching time or interactions.

In this way, the range of decision variables $\bar{x}_1$ and $\bar{x}_2$ should be around $\bar{x}_1^F$ and $\bar{x}_2^S$ with maximum tolerances t_1 and t_2 respectively. The following membership function specify $\bar{x}_1^F$:

$$\mu_{\bar{x}_1}(\bar{x}_1) = \begin{cases} \dfrac{\bar{x}_1 - \left(\bar{x}_1^F - t_1\right)}{t_1} & \bar{x}_1^F - t_1 \le \bar{x}_1 \le \bar{x}_1^F, \\ \dfrac{\left(\bar{x}_1^F + t_1\right) - \bar{x}_1}{t_1} & \bar{x}_1^F \le \bar{x}_1 \le \bar{x}_1^F + t_1, \end{cases} \quad (37)$$

where $\bar{x}_1^F$ is the most preferred solution, the $\left(\bar{x}_1^F - t_1\right)$ and $\left(\bar{x}_1^F + t_1\right)$ is the worst acceptable decision.

Also the following membership function specify $\bar{x}_2^S$:

$$\mu_{\bar{x}_2}(\bar{x}_2) = \begin{cases} \dfrac{\bar{x}_2 - \left(\bar{x}_2^S - t_2\right)}{t_2} & \bar{x}_2^S - t_2 \le \bar{x}_2 \le \bar{x}_2^S, \\ \dfrac{\left(\bar{x}_2^S + t_2\right) - \bar{x}_2}{t_2} & \bar{x}_2^S \le \bar{x}_2 \le \bar{x}_2^S + t_2, \end{cases} \quad (38)$$

where $\bar{x}_2^S$ is the most preferred solution, the $\left(\bar{x}_2^S - t_2\right)$ and $\left(\bar{x}_2^S + t_2\right)$ is the worst acceptable decision.

First, the FLDM goals may reasonably consider $f_1 \ge f_1^F$ is absolutely acceptable and $f_1 < f_1^{\backslash}$ absolutely unacceptable where $f_1^{\backslash} = f_1\left(\bar{x}_1^S, \bar{x}_2^S, \bar{x}_3^S\right)$ and that the preference with $\left[f_1^{\backslash}, f_1^F\right]$ is linearly increasing.

Then the following membership function of the FLDM can be stated as:

$$\mu_{f_1}^{\backslash}\left[f_1(\bar{x})\right] = \begin{cases} 0 & f_1(\bar{x}) \le f_1^{\backslash}(\bar{x}), \\ \dfrac{f_1(\bar{x}) - f_1^{\backslash}(\bar{x})}{f_1^F(\bar{x}) - f_1^{\backslash}(\bar{x})} & f_1^{\backslash}(\bar{x}) \le f_1(\bar{x}) \le f_1^F(\bar{x}), \\ 1 & f_1(\bar{x}) \ge f_1^F(\bar{x}). \end{cases} \quad (39)$$

Second, the SLDM goals may reasonably consider $f_2 \ge f_2^S$ is absolutely acceptable and $f_2 < f_2^{\backslash}$ absolutely unacceptable where $f_2^{\backslash} = f_2\left(\bar{x}_1^T, \bar{x}_2^T, \bar{x}_3^T\right)$ and that the preference with $\left[f_2^{\backslash}, f_2^S\right]$ is linearly increasing.

Then the following membership function of the SLDM can be stated as:

$$\mu_{f_2}^{\backslash}\left[f_2(\bar{x})\right] = \begin{cases} 0 & f_2(\bar{x}) \le f_2^{\backslash}(\bar{x}), \\ \dfrac{f_2(\bar{x}) - f_2^{\backslash}(\bar{x})}{f_2^S(\bar{x}) - f_2^{\backslash}(\bar{x})} & f_2^{\backslash}(\bar{x}) \le f_2(\bar{x}) \le f_2^S(\bar{x}), \\ 1 & f_2(\bar{x}) \ge f_2^S(\bar{x}). \end{cases} \quad (40)$$

Third, the TLDM may be willing to build a membership function for his/her objective function as follows:

ISSN: 2350-0557, Volume-3, Issue-4, July-2016

$$\mu_{f_3}^{\backslash}\left[f_3\left(\overline{x}\right)\right]=\begin{cases}0 & f_3\left(\overline{x}\right)\le f_3^{\backslash}\left(\overline{x}\right),\\ \dfrac{f_3\left(\overline{x}\right)-f_3^{\backslash}\left(\overline{x}\right)}{f_3^{T}\left(\overline{x}\right)-f_3^{\backslash}\left(\overline{x}\right)} & f_3^{\backslash}\left(\overline{x}\right)\le f_3\left(\overline{x}\right)\le f_3^{T}\left(\overline{x}\right),\\ 1 & f_3\left(\overline{x}\right)\ge f_3^{T}\left(\overline{x}\right).\end{cases} \qquad (41)$$

where $f_3^{\backslash}=f_3\left(\overline{x}_1^S,\overline{x}_2^S,\overline{x}_3^S\right)$.

Finally, we can generate the satisfactory solution by solving the following Tchebycheff problem.

$$\max \delta \qquad (42)$$

subject to:

$$\frac{\overline{x}_1-\left(\overline{x}_1^F-t_1\right)}{t_1}\ge \delta I \qquad (43)$$

$$\frac{\left(\overline{x}_1^F+t_1\right)-\overline{x}_1}{t_1}\ge \delta I \qquad (44)$$

$$\frac{\overline{x}_2-\left(\overline{x}_2^S-t_2\right)}{t_2}\ge \delta I \qquad (45)$$

$$\frac{\left(\overline{x}_2^S+t_2\right)-\overline{x}_2}{t_2}\ge \delta I \qquad (46)$$

$$\mu_{f_1}^{\backslash}\left[f_1\left(\overline{x}\right)\right]\ge \delta \qquad (47)$$

$$\mu_{f_2}^{\backslash}\left[f_2\left(\overline{x}\right)\right]\ge \delta \qquad (48)$$

$$\mu_{f_3}^{\backslash}\left[f_3\left(\overline{x}\right)\right]\ge \delta \qquad (49)$$

$$\frac{b_i-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}{\left(a_{i1}^R\overline{x}_1+a_{i2}^R\overline{x}_2+a_{i3}^R\overline{x}_3\right)-\left(a_{i1}^L\overline{x}_1+a_{i2}^L\overline{x}_2+a_{i3}^L\overline{x}_3\right)}\ge \lambda_i \quad , \qquad i=1,2,\ldots,m$$

(50)

$$0\le \lambda_i \le 1 \qquad (51)$$

$$d_1,d_2,d_3,d_4,d_5,d_6\ge 0,\quad d_1+d_2=1$$
$$,d_3+d_4=1,\ d_5+d_6=1 \qquad (52)$$

$$\overline{x}_1\ge 0,\ \overline{x}_2\ge 0,\ \overline{x}_3\ge 0 \qquad (53)$$

where δ is the overall satisfaction, and I is the column vector whose elements are equal to 1.

By solving problem (42)-(53) if the FLDM is satisfied with the obtained solution then satisfactory solution is reached. Otherwise, he/she should provide new membership function for the control variable and objective to the SLDM, who also should provide new membership functions for the control variable and objective to the TLDM until a satisfactory solution is reached.

5. NUMERICAL EXAMPLE

To clarify the solution method for TLP problem with interval coefficients in both of the objective function and constraints. Let us consider the following example:

$$\max_{x_1} f_1\left(x_1,x_2,x_3\right)=\left[2,4\right]x_1+\left[3,6\right]x_2+\left[5,9\right]x_3,$$

where x_2,x_3 solve:

$$\max_{x_2} f_2\left(x_1,x_2,x_3\right)=\left[9,14\right]x_1+\left[4,8\right]x_2+\left[2,7\right]x_3,$$

and x_3 solves:

$$\max_{x_3} f_3\left(x_1,x_2,x_3\right)=\left[1,5\right]x_1+\left[11,14\right]x_2$$
$$+\left[6,10\right]x_3,$$

subject to

$$\left[3,6\right]x_1+\left[5,7\right]x_2+\left[2,5\right]x_3\le 9$$
$$\left[1,2\right]x_1+\left[2,7\right]x_2+\left[3,5\right]x_3\le 12$$
$$x_1,x_2,x_3\ge 0$$

For $d_1=0.5,\ d_2=0.5,\ d_3=0.4,\ d_4=0.6,\ d_5=0.3,\ d_6=0.7,$ and $\lambda_1=\lambda_2=0.5$ and by using mathematical treatment described in section 3 the above problem can be formulated as the following deterministic TLP problem:

$$\max_{x_1} f_1\left(x_1,x_2,x_3\right)=2x_1+3x_2+4.5x_3,$$

where x_2,x_3 solve:

$$\max_{x_2} f_2\left(x_1,x_2,x_3\right)=6.1x_1+3.6x_2+3.3x_3,$$

and x_3 solves:

$$\max_{x_3} f_3\left(x_1,x_2,x_3\right)=2.3x_1+4.8x_2+3.8x_3,$$

subject to

$$4.5x_1+6x_2+3.5x_3\le 9$$
$$1.5x_1+4.5x_2+4x_3\le 12$$
$$x_1,x_2,x_3\ge 0$$

By using the fuzzy method proposed in section 4, the solution of the first level is $\left(\overline{x}_1^F,\overline{x}_2^F,\overline{x}_3^F,\alpha^F\right)=\left(0,0,2.571,1\right)$ and $f_1^F=11.569$. The solution of the second level is $\left(\overline{x}_1^S,\overline{x}_2^S,\overline{x}_3^S,\beta^S\right)=\left(2,0,0,1\right)$ and $f_2^S=12.2$. The solution of the third level is

$\left(\overline{x}_1^T, \overline{x}_2^T, \overline{x}_3^T, \gamma^T\right) = \left(0, 0.0008, 2.57, 1\right)$ and $f_3^T = 9.769$.

Let the FLDM control decision $\overline{x}_1^F$ is around 0 with tolerance $t_1 = 1$ and the SLDM control decision $\overline{x}_2^S$ is around 0 with tolerance $t_2 = 1$. Then the Tchebycheff problem (42)-(53) take the following form:

$$\max \delta$$

subject to:

$$x_1 + 1 \geq \delta I$$
$$1 - x_1 \geq \delta I$$
$$x_2 + 1 \geq \delta I$$
$$1 - x_2 \geq \delta I$$
$$2x_1 + 3x_2 + 4.5x_3 - 7.567\delta \geq 4$$
$$6.1x_1 + 3.6x_2 + 3.3x_3 - 3.72\delta \geq 8.48$$
$$2.3x_1 + 4.8x_2 + 3.8x_3 - 5.169\delta \geq 4.6$$
$$4.5x_1 + 6x_2 + 3.5x_3 \leq 9$$
$$1.5x_1 + 4.5x_2 + 4x_3 \leq 12$$
$$x_1, x_2, x_3 \geq 0$$
$$\delta \in [0,1]$$

Whose solution is:

$$\left(x_1, x_2, x_3\right) = \left(0.666, 0, 1.7152\right), \quad \delta = 0.334,$$
$$f_1 = 9.0504, f_2 = 9.7228 \text{ and } f_3 = 8.0496.$$

6. CONCLUSION

A solution method for TLP problem with interval coefficients has been introduced. These coefficients are located in both of objective functions and constraints. A fuzzy approach is used in the introduced solution method. The concept of tolerance membership function is used. The investigation of other solution methods for multi-level programming with interval coefficients would be future topics.

REFERENCES

[1] Abo-Sinna, M.A., 2001. A bi-level non-linear multi-objective decision-making under Fuzziness. Journal of Operational Research Society (OPSEARCH), 38 (5), 484–495.

[2] Anandolingam, G., 1988. A mathematical programming model of decentralized multi-level system. Journal of Operational Research Society, 39, 1021–1033.

[3] Anandalingam, G. and Apprey, V., 1991. Multi-level programming and conflict resolution. European Journal of Operational Research, 51, 233–247.

[4] Bialas, W.F. and Karwan, M.H., 1984. Two-level linear programming, Management Science, 30, 1004–1020.

[5] Cassidy, R., Kirby, M. and Raike, W., 1971. Efficient distribution of resources through three levels of government. Management Science, 17, 462–473.

[6] Emam, O., El-Araby, E. M. and Belal, M. A., 2015. On Rough Multi-Level Linear Programming Problem. Information Sciences Letters, 4 (1), 41-49.

[7] Hsu, S.S., Lai, Y.J. and Stanley, L.E., 2001. Fuzzy and Multi Level Decision-Making. Springer-Verlag Ltd, London.

[8] Hu, Y.D., 1990. Applied Multiobjective Optimization. Shanghai Science and Technology Press. Shanghai. China.

[9] Inuiguchi, M. and Kume, Y., 1991. Goal Programming Problems with interval coefficients and target intervals. European Journal of Operational Research, 52, 345-360.

[10] Jiang, C., Han, X., Liu, G.R. and Liu, G.P., 2008. A nonlinear interval number programming method for uncertain optimization problems. European Journal of Operational Research, 188, 1–13.

[11] Olivera, C. and Antunes, C.H., 2007. Multiple objective Linear Programming Models with Interval Coefficients – an illustrated overview. European Journal Of Operational Research, 181, 1434-1463.

[12] Osman, M.S., Abo-Sinna, M.A., Amer, A.H. and Emam, O.E., 2004. A multi-level non-linear multi-objective decision-making under fuzziness. Applied Mathematics and Computation, 153, 239–252.

[13] Sakawa, M., 1993. Fuzzy Sets and Interactive Multi-Objective Optimization. New York : Plenum Press.

[14] Shi, X. and Xia, H., 1997. Interactive bi-level multi-objective decision-making. Journal of Operational Research Society, 48, 943–949.

[15] Sinha, S., 2003. Fuzzy mathematical Programming applied to multi-level programming problems. Computers and Operations Research, 30, 1259-1268.

[16] Wen, U.P. and Bialas, W.F., 1986. The hybrid algorithm for solving the three-level linear programming problem. Computers and Operations Research, 13, 367–377.

