# Mathematical Modeling and Optimization of a Parabolic Trough Concentrator for the Improvement of Collection Efficiency

**Arun Kumar**
Centre for Energy Resources Development, Department of Mechanical Engineering, Indian Institute of Technology, (Banaras Hindu University),
Varanasi – 221005, India

**S.K.Shukla**
Centre for Energy Resources Development, Department of Mechanical Engineering, Indian Institute of Technology, (Banaras Hindu University),
Varanasi – 221005, India

## ABSTRACT

In the present scenario of a huge energy demand, dependency on fossil fuels only, certainly creates crisis in future especially for developing country. Although renewable resources of energy like solar energy is being utilized on a broad scale now a days but the problem comes in law and economy i.e. social and acceptability. Economy is the main criteria in modern era of huge energy requirement and its social acceptability. Keeping these points in view, a concentrating unit i.e. parabolic trough concentrator has been designed in such a way that it can produce optimum power. An empirical relation has been established between receiver area and aperture area and it was found that concentration ratio, CR is 81.69.Ratio of the height of the parabola to the focal length (**σ**) is 1.7 and between aperture diameter, D and focal length, f is D = 5.21 f. The value of rim angle corresponding to the optimum value of concentration ratio 81.69 is 74.97°. These optimized values which have been calculated in this paper, is able to achieve the maximum collection efficiency of a parabolic trough concentrator.

## Keywords

Parabolic trough concentrator; Concentration ratio; Collection efficiency; Helical coil solar cavity receiver

## 1. INTRODUCTION

Parabolic trough concentrator is a line focus collector and is applicable where medium brightness and radiation balance concentrations are sufficient [1]. Global warming induced due to co2, has become stressing issue and need to be tacle. Proper utilization renewable energy resources, especially solar energy, is increasingly being considered as a promising solution to global warming and means for achieving sustainable development for mankind. The sun releases an enormous amount of solar radiation to its surroundings out of which 174 PW (1PW=1015 W) amount of radiation is reached at the upper surface of atmosphere of the earth. During the journey from upper atmosphere to the surface of the earth, the radiation attenuated twice by both the atmosphere and the clouds. Attenuation through the atmosphere is, 6% by reflection and 16% by absorption, and through the clouds, 20% by reflection and 3% by absorption. And rest of the total incoming radiation (51 PW) reaches the land and the ocean [2].

John Ericssion, [3] constructed the first knownparabolic trough concentrator. They used it to power a hot air engine. Germans Wilhelm Meier and Adolf Remshardt [4] obtained the first patent of parabolic trough technology. The purpose behind it was generation of steam.

The English F. Shuman and the American C.V. Boys [4] built the world's first solar thermal power station in Meadi, Egypt between 1912 and 1913.

Bala Bhaskar [5] In india first solar power plant of 50 kW capacity was installed by NES using the parabolic trough technology (line focusing) at the Gwal Pahari.it was commissioned in 1989 and operated till 1990.A Solar Thermal Power Plant of 140MW at Mathania in Rajasthan, has been proposed and sanctioned by the Government in Rajasthan.

Ari Rabl [6] compared various solar concentrators in terms of general characteristics, named as number of reflections varies with angle of incidence, concentration, acceptance angle, sensitivity to mirror errors, size of reflector area and average number of reflections for the development of a new solar concentrator.. The connection between concentration, acceptance angle and operating temperature of a solar collector is analyzed in simple intuitive terms for designing collectors with maximal concentration. They proposed some new concentrators including the use of compound parabolic concentrators as second stage concentrators for conventional parabolic or Fresnel mirrors. Such a combination approaches the performance of an ideal concentrator without demanding a large reflector.

Valentina A. Salomoni et al. [7] has given a general view related to the last experiences R&D in the field of new technologies for solar energy exploitation within the Italian context. One of the most relevant objectives of this research program is the study of CSP systems operating in the field of medium temperatures (about 550°C), directed towards the development of a new and low-cost technology to concentrate the direct radiation and efficiently convert solar, and another aspect is focused on the production of hydrogen by means of thermo-chemical processes at temperatures above 800°C. Within the medium temperature field, an innovative approach has been presented for the conceptual design of liquid salts concrete storage systems. A multi-tank sensible-heat storage system has been proposed for storing thermal energy, with a two tanks molten salt system.

Joshua FOLARANMI [8] reports the design, construction and testing of a parabolic dish solar steam generator It also describes the sun tracking system unit by manual tilting of the lever at the base of the parabolic dish to capture solar energy. On the average sunny and cloud free days, the test results gave high temperature above 200°C. Temperature above 200°C was obtained at base of the absorber. Water boiled faster using the generator than when using ordinary charcoal or kerosene stove. The parabolic dish solar steam generator is very efficient heating equipment.

Ming Qu et al. [9] developed heat transfer and mass balance PTSC model which is very useful for solar heating and cooling system design. It not only helps to select the proper operating conditions and to detect the possible problems in the design of system, but also provides some basic figures for device's selection like pump as well and they calculated single dimensional heat transfer such radiative, convective, conductive and mass and energy that is solved by engineering equation solver. It plotted number of graph between operating parameters.

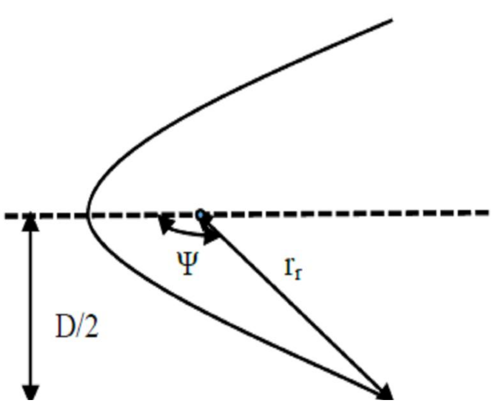
Scott A. Jones et al. [10] created a model of the 30 MWe SEGS VI parabolic trough plant was created in the TRNSYS simulation environment using the Solar Thermal Electric Component model library. it demonstrates the capability to perform detailed analysis and is useful for such things as evaluating proposed trough storage systems. This model could be used as the basis for evaluating the annual performance and operational issues of proposed trough thermocline storage concepts, including the effects of startup, shutdown, and cloud transients.

# 2. MATHEMATICAL ANALYSIS

## 2.1 Assumptions

Following are the assumptions for mathematical calculations,

- Total solar radiation intensity after reflection from the aperture of a parabolic trough concentrator is intercepted by receiver at the focus.
- No portion of aperture area of a parabolic trough concentrator is shaded by the receiver.
- Solar radiation intensity falls on the aperture of a parabolic trough concentrator is constant.



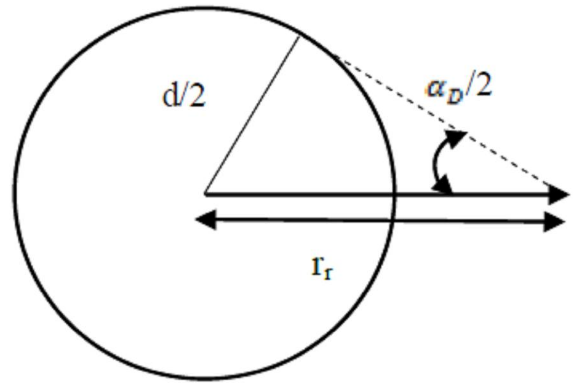




Figure – 1(b) Receiver parameters

From figure 3(b) we can write from right angled triangle

$$d/2 = r_r \sin(\alpha_D/2) \quad (1)[11]$$

$$d = 2 \times r_r \sin(\alpha_D/2) \quad (2)$$

Alternatively we can write $r_r$ in terms of D and $\psi$

$$r_r = \frac{D}{2\sin\psi} \quad (3)$$

From (2) and (3) we can conclude that,

$$d = \frac{D}{\sin\psi}(\sin\alpha_D/2) \quad (4)$$

We can express the rim angle and terms of d and h

$$\tan\Psi_{rim} = \frac{1}{\left(\frac{d}{8h}\right)-\left(\frac{2h}{d}\right)} \quad (5)[12]$$

$$\tan\Psi_{rim} = \frac{\left(\frac{f}{d}\right)}{2\left(\frac{f}{d}\right)^2-\left(\frac{1}{8}\right)} \quad (6)[12]$$

Relationship between h, f & D

$$h = \frac{D^2}{16f} \quad (7)[13]$$

Now the arc length of parabolic trough can be calculated as,

As we know the general formula for calculating arc length is

Equation of parabola is $y^2 = 4fx$ (8)

So, $x = \frac{y^2}{4f}$

or we can also write $f(y) = \frac{y^2}{4f}$

Differentiating it with respect to y

$$f'(y) = \frac{y}{2f} \tag{9}$$

As we know General formula for calculating the arc length

$$s = \int_{y_1}^{y_2} \sqrt{1 + (f'(y))^2}\, dy \qquad (10)[14]$$

Here limits are $y_1 = -D/2$ & $y_2 = +D/2$

$$s = \int_{-D/2}^{+D/2} \sqrt{1 + (f'(y))^2}\, dy$$

Therefore, after integration we have,

$$s = \frac{D}{2}\sqrt{1 + \frac{D^2}{16f^2}} + 2f \times \ln\left(\frac{D}{4f} + \sqrt{1 + \frac{D^2}{16f^2}}\right) \tag{11}$$

Now as we know from equation (7) $h = \frac{D^2}{16f}$

So from this relation we can get arc length in terms of $h$ and $D$ by eliminating $f$ in terms of $h$ and D.

$$\frac{4h}{D} = \frac{D}{4f}$$

So,

$$s = \frac{D}{2}\sqrt{1 + \left(\frac{4h}{D}\right)^2} + 2f \times \ln\left(\frac{4h}{D} + \sqrt{1 + \left(\frac{4h}{D}\right)^2}\right) \tag{12}$$

We can calculate the area of parabolic trough aperture as follows

$$A_a = s \times L \tag{13}$$

Where L is the length of the parabolic trough and s is the arc length from calculated as above

And likewise we can calculate the area of cylindrical absorber as follows

$$A_r = \pi \times d \times L \tag{14}$$

So we can calculate concentration ratio (C.R.) as per definition

$$C.R. = \frac{A_a}{A_r} \tag{15}$$

After putting the values of $A_r$ and $A_a$

$$C.R. = \frac{s}{\pi \times d}$$

$$\frac{A_a}{A_r} = \frac{s}{\pi \times d}$$

$$A_a = \frac{s}{\pi \times d} \times A_r$$

Now from the equation (12) after putting the value of 's' we get

$$A_a = \frac{\frac{D}{2}\sqrt{1+\left(\frac{4h}{D}\right)^2} + 2f \times \ln\left(\frac{4h}{D} + \sqrt{1+\left(\frac{4h}{D}\right)^2}\right)}{\pi \times d} \times A_r \tag{16}$$

Now from equation (4),

$$d = \frac{D}{\sin\psi}\left(\sin\frac{\alpha_D}{2}\right)$$

Where $\alpha_D$ = sun beam angle, (32´)

After putting value of $\alpha_D$, we obtain

$$d = \frac{D}{\sin\psi}(0.004654)$$

Now putting this value of d in equation (16)

$$A_a = \frac{\left[\frac{D}{2}\sqrt{1+\left(\frac{4h}{D}\right)^2} + 2f \times \ln\left(\frac{4h}{D} + \sqrt{1+\left(\frac{4h}{D}\right)^2}\right)\right]}{\pi \times \frac{D}{\sin\psi}(0.004654)} \times A_r$$

$$A_a = \frac{\sin\psi\left[\frac{1}{2}\sqrt{1+\left(\frac{4h}{D}\right)^2} + \frac{2f}{D} \times \ln\left(\frac{4h}{D} + \sqrt{1+\left(\frac{4h}{D}\right)^2}\right)\right]}{(0.01461417)} \times A_r$$

$$A_a = \sin\psi\left[\begin{array}{c} 34.213\sqrt{1+\left(\frac{4h}{D}\right)^2} + \frac{136.8534\ f}{D} \times \\ \ln\left(\frac{4h}{D} + \sqrt{1+\left(\frac{4h}{D}\right)^2}\right) \end{array}\right] \times A_r$$

$$A_a = \frac{\sin\psi}{D}\left[\begin{array}{c} 34.2133\sqrt{D^2 + 16h^2} + \\ 136.8534\ f \times \ln\left(\frac{4h + \sqrt{D^2 + 16h^2}}{D}\right) \end{array}\right] \times A_r \tag{17}$$

Now from equation (7) we have,

$h = \frac{D^2}{16f}$ We can calculate D in terms of $h$ and $f$

$$D = \sqrt{16hf} = 4\sqrt{hf} \tag{18}$$

Now putting this value of D in equation (17)

$$A_a = \frac{\sin\psi}{4\sqrt{hf}}\left[\begin{array}{c} 34.2133\sqrt{16hf + 16h^2} + \\ 136.8534\ f \times \ln\left(\frac{4h + \sqrt{16hf + 16h^2}}{4\sqrt{hf}}\right) \end{array}\right] \times A_r$$

$$A_a = \frac{\sin\psi}{4\sqrt{hf}}\left[\begin{array}{c}136.8532\sqrt{h(h+f)} + \\ 136.8534\, f \times \ln\left(\frac{h+\sqrt{h(h+f)}}{\sqrt{hf}}\right)\end{array}\right] \times A_r \qquad (19)$$

As we know from the equation (6)

$$\tan \Psi_{rim} = \frac{\left(\frac{f}{D}\right)}{2\left(\frac{f}{D}\right)^2 - \left(\frac{1}{8}\right)}$$

Let's take $\left(\frac{f}{D}\right) = k$ for ease of calculation and putting this in equation (6) to get

$\tan \Psi_{rim}$ in terms of k

$$\tan \Psi_{rim} = \frac{(8k)}{16k^2 - 1}$$

Now From the Pythagoras theorem we can calculate the value of $\sin\Psi_{rim}$

$$\sin\Psi_{rim} = \frac{8k}{\sqrt{(8k)^2 + (16k^2-1)^2}}$$

$$\sin\Psi_{rim} = \frac{8k}{\sqrt{(16k^2+1)^2}} \qquad (20)$$

Simplification $[(8k)^2 + (16k^2 - 1)^2 = (16k^2 + 1)^2$

$$\sin\Psi_{rim} = \frac{8k}{(16k^2+1)} \qquad (21)$$

Now putting value $\left(\frac{f}{D}\right) = k$

$$\sin\Psi_{rim} = \frac{8\left(\frac{f}{D}\right)}{\left[16\left(\frac{f}{D}\right)^2 + 1\right]} \qquad (22)$$

As from equation (18)

$$D = \sqrt{16hf} = 4\sqrt{hf}$$

Putting the value of D in equation (22) we get

$$\sin\Psi_{rim} = \frac{8\left(\frac{f}{4\sqrt{hf}}\right)}{\left[16\left(\frac{f}{4\sqrt{hf}}\right)^2 + 1\right]}$$

$$\sin\Psi_{rim} = \frac{\left(\frac{2f}{\sqrt{hf}}\right)}{\left[\frac{f^2}{hf} + 1\right]} \qquad (23)$$

Upon simplifying equation (23)

$$\sin\Psi_{rim} = \frac{2\times\sqrt{hf}}{h+f} \qquad (24)$$

Now putting this value of $\sin\Psi_{rim}$ in equation (19)

$$A_a = \frac{\sin\psi}{D}\left[\begin{array}{c}34.2133\sqrt{D^2+16h^2} + \\ 136.8534\, f \times \ln\left(\frac{4h+\sqrt{D^2+16h^2}}{D}\right)\end{array}\right] \times A_r$$

$$A_a = \frac{136.8532}{2(h+f)}\left[\sqrt{h(h+f)} + f \times \ln\left(\frac{h+\sqrt{h(h+f)}}{\sqrt{hf}}\right)\right] \times A_r$$

$$A_a = \frac{136.8532}{2(h+f)}\left[\sqrt{h(h+f)} + f \times \ln\left(\sqrt{\frac{h}{f}} + \sqrt{1+\frac{h}{f}}\right)\right] \times A_r$$

$$A_a = \frac{68.4266}{(1+\frac{h}{f})}\left[\sqrt{\frac{h}{f}\left(1+\frac{h}{f}\right)} + \ln\left(\sqrt{\frac{h}{f}} + \sqrt{1+\frac{h}{f}}\right)\right] \times A_r \qquad (25)$$

Now defining a new variable "$\sigma$" $= \frac{h}{f}$ and inserting it in place of $\frac{h}{f}$ in equation (25)

$$A_a = \frac{68.4266}{(1+\sigma)}\left[\sqrt{\sigma(1+\sigma)} + \ln\left(\sqrt{\sigma} + \sqrt{1+\sigma}\right)\right] \times A_r \qquad (26)$$

$$A_a = 68.4266\left[\sqrt{\frac{\sigma}{1+\sigma}} + \frac{1}{1+\sigma}\ln\left(\sqrt{\sigma} + \sqrt{1+\sigma}\right)\right] \times A_r \qquad (27)$$

Here $A_a$ and $A_r$ are related to only a single variable which is "$\sigma$"

$$\mathbf{C.R.} = \frac{A_a}{A_r} = \mathbf{68.4266}\left[\begin{array}{c}\sqrt{\frac{\sigma}{1+\sigma}} + \\ \frac{1}{1+\sigma}\,\mathbf{ln}\left(\sqrt{\boldsymbol{\sigma}} + \sqrt{\mathbf{1}+\boldsymbol{\sigma}}\right)\end{array}\right] \qquad (28)$$

Here, C.R. $= f(\sigma)$, therefore, to obtain the optimum value of concentration ratio, the differential of C.R. with respect to $\sigma$ must be equal to zero.

$\frac{d(C.R.)}{d\sigma} = 0$, we can see the variation of concentration ratio with respect to $\sigma$ as shown in figure-6. From this curve we can see when $\sigma = 1.7$ value of concentration ratio is 81.69.

or $\frac{A_a}{A_r} = 81$

Another empirical relation between height of the parabola and its focal length is h = 1.7 f and between aperture diameter, D and focal length, f is D = 5.21 f. The value of rim angle corresponding to the optimum value of concentration ratio 81.69 is 74.97°.

## 3. RESULTS AND DISCUSSION

Results shown below which gives the clue for getting the optimum value of rim angle corresponding to have maximum collection efficiency, receiver diameter must be greater than the focal length. Figure-2(a) shows that the focal length is greater than that of receiver diameter and in this case optimum value of rim angle has not been achieved.

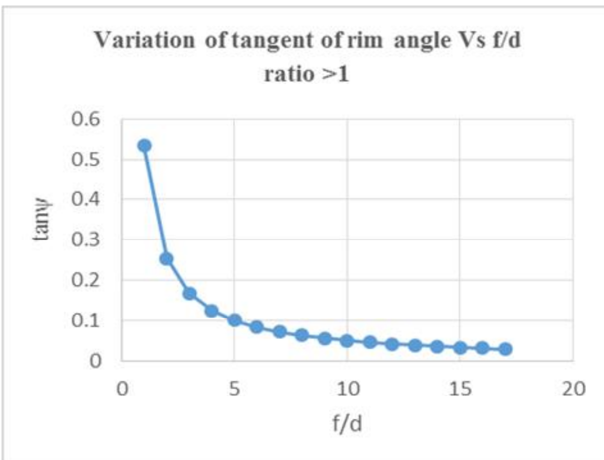



Figure-2(a) shows the variation of tangent of rim angle with respect to f/d ratio, greater than 1.

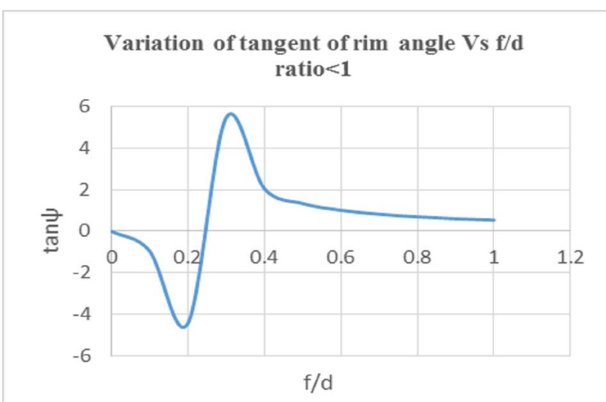



Figure-2(b) shows the variation of tangent of rim angle with respect to f/d ratio, less than 1.

But figure-2(b) shows that the value of receiver diameter which is greater than that of the focal length and in this case the optimum value of rim angle which is equal to 74.97° is achieved. So, one must have to proceed according to results shown in figure-2(b).

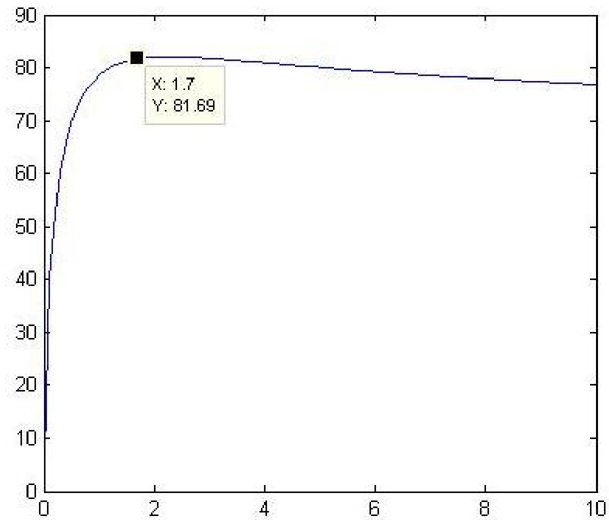



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Figure -3 represents the variation of concentration ratio with '$\sigma$ (= h/f)'. As the value of $\sigma$ increases, the value of concentration ratio increases rapidly and reaches a constant value of 81.69 at $\sigma$ = 1.7 and if we further increase the value of $\sigma$, then the value of concentration ratio either remains constant or decreasing with very low rate. Hence for a achieving the maximum value of collection efficiency together with reducing the cost of material it is essential to take the value of concentration ratio which is equal to 81.69.

## 4. CONCLUSION

On the whole we come to the conclusion that the optimum value of concentration ratio for parabolic trough concentrator is 81 and the ratio of height to the focal length is 1.7. a plot has been drawn between concentration ratio and height to focal length ratio, from this plot it is clear that if the value of height to focal length ratio further increase greater than '2' then the value of concentration ratio remains almost constant and equal to 81.69. It means if the height to focal length ratio will have been taken more than two then it is only the wastage of material and we have to pay a lot. According to D.Canabarro et. al. [15] receiver length and pipe length are determined on the basis of aperture size of PTs and one of the way to reduce the cost is to increase the aperture size. The standard troughs are designed for maximum concentration i.e. rim angle close to $90^0$.This results in the Centre of mass of parabolic trough receiver system is very far from the Centre of receiver.

## 5. ACKNOWLEDGE

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## 6. NOMENCLATURE

$A_a$ : Aperture area of parabolic trough
$A_r$ : Absorber area of cylindrical receiver
C.R. :Concentration ratio
($C.R._o$) : Optical concentration ratio
($CR_g$ or $CR$ ) : Geometric concentration ratio
D :Aperture diameter
d : Absorber diameter
h : Depth of the parabola
f : Focal length
$\alpha_D$ : Sun beam angle, (32´)
$r_r$ : Distance between the absorber tube and the mirror rim
$\Psi / \Psi_{rim}$ : Rim angle
$\gamma$ : Intercept factor
$I_a$ : Insolation
$I_r$ : Irradiance
s : Arc length of the parabola

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