

Modeling and Forecasting Market Value-at-Risk of DS30 Index through GARCH Family Models with Heavy Tail Distribution

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ABSTRACT

In light of the latest global financial crisis and the ongoing sovereign debt crisis, accurate measuring of market losses has become a very current issue. One of the most popular risk measures is Value-at-Risk (VaR). A set of symmetric and asymmetric GARCH type models based on various error distributions were applied on Dhaka Stock Exchange DS30 Index from January 28, 2013 to May 29, 2017 for estimating and forecasting the market Value-at-Risk of the index. The most adequate GARCH family models for estimating volatility in the Dhaka stock exchange was found to be as the asymmetric TGARCH (1,1) model with GED. TGARCH (1,1) model with GED was allowed by Kupiec test with 99% of confidence level. The proposed VaR model would help the investors in their emerging capital markets.

Keywords

Value-at-Risk, GARCH-type models, backtesting, DS30 and Heavy tail distribution.

1. INTRODUCTION

Value-at-risk, as defined by P Jorion is “the worst loss over a target horizon with a given level of target probability”. P Jorion, Value-at-Risk: the Benchmark for Controlling Market risk, McGraw-Hill Professional Book Group. From a mathematical purpose of read, Value-at-Risk is simply a quantile of a return distribution function. The portfolio’s Value-at-Risk (VaR) may be a grade of its return distribution over a hard and fast horizon. Value-at-Risk (VaR) may be a risk calculation tool that relies on loss distributions. This risk was measured by few authors (Morgan, 1994; Linsmeier and Pearson, 1996).

The estimation of VaR can be performed in different ways, e.g., historical, simulation, extreme value theory, modelling and so on (Brooks & Persaud, 2003; Angelidis et al., 2004; Orhan and Köksal, 2012). One of the problems to estimate VaR in case of modelling is the volatility. For this, GARCH family models might be used as the remedial method (Engle et al., 1987; Nelson, 1991; Glosten et al., 1993; Zakoian, 1994; Schwert, 1989; Ding et al., 1993; French et al., 1987; Chou, 1988). For indices of Dhaka stock exchange, researchers have been working in many dimensions (Basher et al., 2007; Chowdhury, 1994, Hassan and Maroney, 2004; Chowdhury et al., 2006; Mulla, 2009; Hossain

and Uddin, 2011; Rayhan et al., 2011; Islam et al., 2012; Alam et al., 2013; Mukit, 2013; Islam et al., 2014 and so on).

Dhaka Stock Exchange DS30 index is the aggregate of top A category 30 indices. To the best of our knowledge, research on DS30 index is yet undone, moreover, VaR for DS30 index is also rare. The reason is that the non-normality and heavy tail of the heteroscedastic error distribution. Unreal facts of the securities market can be detected and discarded by the method of Heavy-tail distributions but generally GARCH models with heavy-tail distributions are comparatively higher fitted to analyzing returns on stocks (Alberg et al., 2006). The non-normality of the residual might be handled with different method, e.g., student-t, skewed-t, and generalized error distributions (Bollerslev, 1986; Engle, 1982; Nelson, 1991; Harvey, 1981; Hung-Chun and Jui-Cheng, 2010; Theodossiou, 1998 and so on). Thus, the purpose of the present study is to model and forecast the market Value-at-Risk of DS30 index through GARCH family models with heavy tail distribution.

2. METHOD

The number of relevant risk factors can be terribly huge, probably reaching the thousands. We have a tendency to specialize in the additional elementary issue of activity the tail of the loss distribution, notably at giant losses. A target probability provides an easy manner of summarizing info regarding the tail of the loss distribution, and this explicit worth of the target probability is commonly taken as an inexpensive worst-case loss level. The importance of Value-at-Risk lies in its specialize in the tail of the loss distribution. Before estimating VaR we will estimate GARCH family models using the suitable software, the models to forecast the volatility. In consequence of these results, we have a tendency to settle for that the most effective model using the Akaike information Criterion (AIC), Bayesian Information Criterion (BIC), Hannan-Quinn (HQ) and log likelihood value. The DS30 index from Dhaka Stock Exchange series sample starts in 28 January 2013 and ends in 29 May 2017 for the following considerations.

Historical simulation (HS) may be a non-parametric and unconditional technique. Its main advantage is that it's straightforward to implement and needs few assumptions. The key assumption is that the distribution of losses (or returns) within the portfolio is constant over the sample period and may be a decent predictor of forthcoming behavior. For this technique, i.e. if we have a tendency to invest in an exceedingly straightforward stock

or a portfolio, we have a tendency to take the historical returns for the last (say) a hundred days and type from the worst to the most effective return. If you would like to form the assessment for a confidence interval, i.e. 95% we have a tendency to state that the VaR 95% is adequate fifth lowest return. The interpretation of this value is “with a probability 95% the loss for ensuing day won't be worse than the fifth lowest return”. However, we must always take care once applying this technique as a result of it's terribly sensitive to the length of data because the sample might not be a decent representative of the right distribution (Dowd, 1998).

VaR can be viewed as a gauge that summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence (Jorion, 2007).

More formally, a (α) VaR is expressed as

$$\Pr(L > VaR) = \alpha$$

where L is the loss on a given day and α is the significance level. VaR is therefore a *quantile* in the distribution of profit and loss that is expected to be exceeded only with a certain probability, formally expressed as

$$p = \int_{-\infty}^{-VaR(p)} f_q(x) dx$$

Throughout this thesis, the VaR figures will be given using a 1 % and 5 % significance level, i.e. 1 % and 5 % VaR estimates will be presented.

VaR is computed using the conditional volatility of returns multiplied by the quantile of a given probability distribution, e.g. the Normal distribution as shown in Equation below:

$$(\alpha)VaR = -\sigma_t \phi_\alpha$$

where ϕ_α in the Normal distribution is equal to -2,33 for a 1 % VaR and -1,65 for a 5 % VaR. Thus, VaR is presented as a positive number.

2.1 Backtesting VaR

Finding appropriate forecast models for VaR estimates needs a technique for assessing the predictions ex-post. The VaR estimates during this thesis are calculated by two checks: an unconditional and a conditional test of coverage originally developed by Kupiec (1995) and Christoffersen (1998) severally.

Hereafter, daily return are tagged consistent with Equation (21) so as to outline whether or not the daily return exceeded the VaR estimate or not. The indicator variable is built as

$$\eta_t = \begin{cases} 1 & \text{if } r_t < -VaR \\ 0 & \text{if } r_t \geq -VaR \end{cases}$$

where 1 specifies a violation and 0 specifies a return lower than the VaR. The violations are thenceforth summed and divided by the whole range of out-of-sample VaR estimates with the intention of getting the empirical size.

2.2 Kupiec's test

Kupiec's test was established to test whether or not the experiential proportion of violations congregate with the nominal proportion fixed by the VaR significance level. Kupiec (1995) recommends a likelihood ratio test created as in Equation below.

$$LR_{uc} = 2 \ln \left[\left(1 - \frac{F}{T} \right)^{T-F} \left(\frac{F}{T} \right)^F \right] - 2 \ln [(1 - p)^{T-F} p^F]$$

where T is the number of out-of-sample estimates and F the observed number of violations. Hence, F/T is the empirical VaR size which follows the binomial distribution so $F \sim B(T, p)$. LR_{uc} follows the chi-square distribution with one degree of freedom, i.e. $LR_{uc} \sim \chi^2_{(1)}$, under the null hypothesis which states that $F/T = p$. Hence, a rejection of the null hypothesis implies that the empirical VaR size is significantly different from the stated VaR significance level, i.e. the nominal size.

3. RESULTS

The DS30 index series sample starts in 28 January 2013 and ends in 29 May 2017 for the following considerations. An initial investigation of data exhibits some indications about Dhaka stock market. In contrast to the identical Gaussian distribution, literature suggests that the distribution of stock returns exhibit the following features: skewness, leptokurtosis and volatility. For each observation, the simple daily stock log-returns at time t were calculated.

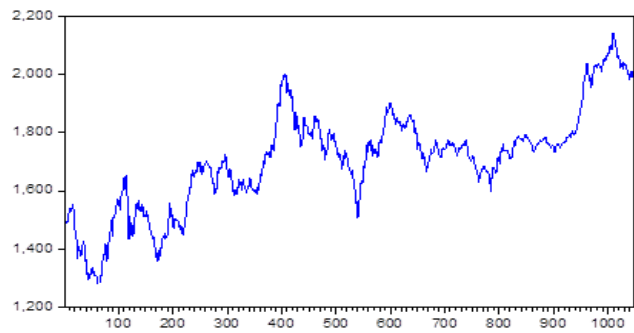


Figure 1. Time series plot of daily DS30 index

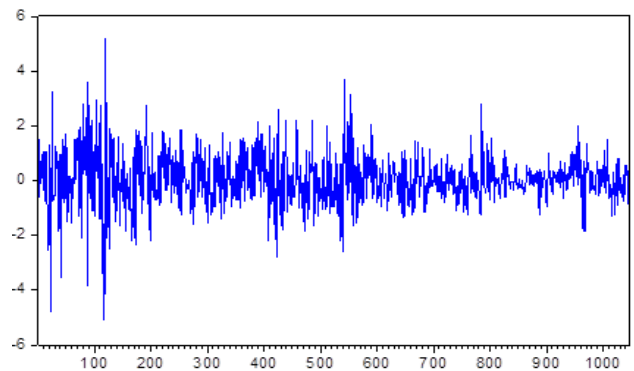


Figure 2. Daily returns of DS30 index

Descriptive measures and histogram of DS30 index return series is displayed in Figure 3 negative skewness of returns were exhibited for DS30, and excess kurtosis also observed for the series. Normality tests (Jarque-Bera test) were rejected ($\alpha=1\%$); the unit root tests (ADF and PP test) in Table 1 suggested that the return series is stationary, which is consistent with most financial time series data.

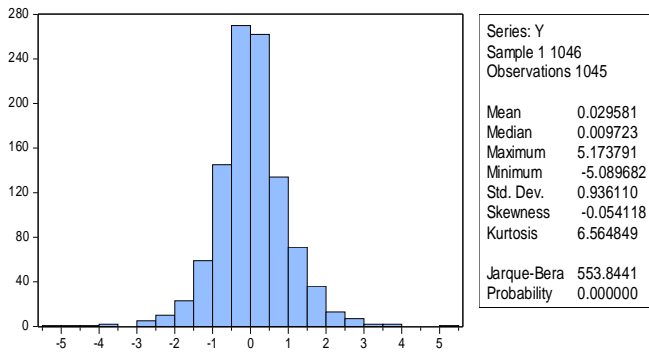


Figure 3. Histogram and descriptive statistics of DS30 returns

Original data series is displayed in Figures 1 and volatility clustering is displayed in Figure 2 for the DS30 return index. In the Figure 2, it is noticeable that periods of low volatility are followed by periods of low volatility for a prolonged period, and periods of high volatility are followed by periods of high volatility for a prolonged period, which justifies the applications of ARCH family models.

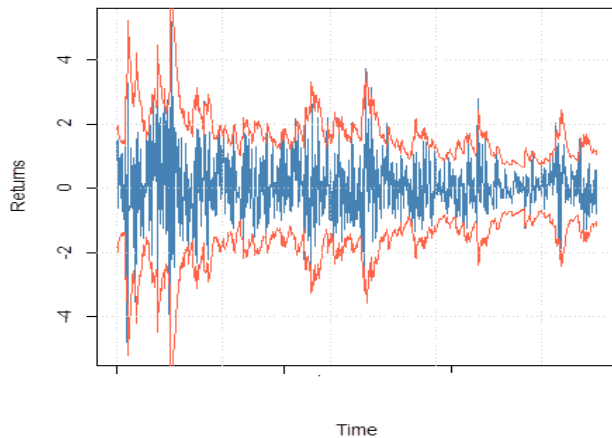


Figure 4. Series with 2 conditional standard deviation.

Table 1 shows the ARCH effect as formulated by Engle (1982). In Table 1 show signs of heteroscedasticity in DS30 index; this indicates the necessity of using some other ARCH family models. The ARCH model was estimated for daily returns index using the ML - ARCH (Marquardt) - Normal distribution. The AIC, BIC information criteria and a set of model diagnostic tests such as ARCH-LM test were applied to choose the volatility models which represent the conditional variance appropriately.

Table 1. Unit root test and ARCH-LM test of DS30 return series.

Value	ADF	PP
t- statistic	-28.18617	-27.84357
p value	0.0000	0.0000
ARCH-LM test statistic	179.36	
p value	0.0000	

Table 2 gives results parameters estimation, AIC BIC, Hannan-Quinn, log likelihood value and ARCH-LM test. Several GARCH models with symmetry and asymmetry models were fitted but selected four models was presented in the table. From the estimated models TGARCH (1,1) with GED is better performing by using AIC BIC, HQ and log likelihood value. And ARCH-LM test suggest that there is no remaining ARCH effect.

After estimating the different GARCH-type models, we obtained the standardized residuals from these models to carry out the diagnosis tests to establish the goodness of fit of these alternative models. In theory, the standardized residuals are expected to have a mean of zero and a variance of unity.

Table 2. Maximum Likelihood estimates of several GARCH models.

Parameters	GARCH(1,1) GED	GARCH(1,1) NOR	TGARCH(1,1) NOR	TGARCH(1,1) GED
Variance Model				
ω (constant)	0.01015	0.01191	0.01238	0.01098
α (ARCH effect)	0.14511	0.15133	0.11464	0.11088
β (GARCH effect)	0.85160	0.84315	0.84464	0.84897
γ (Leverage effect)	-	-	0.07357	0.07805
Log likelihood	-1249.659	-1257.572	-1253.856	-1246.387
AIC	2.3993	2.4126	2.4074	2.3950
BIC	2.4183	2.4268	2.4263	2.4157
HQ	2.4065	2.4180	2.4146	2.4040
ARCH-LM statistics	0.08806	0.05132	0.0000688	2.876e-05
p value	0.7667	0.8208	0.9934	0.9957

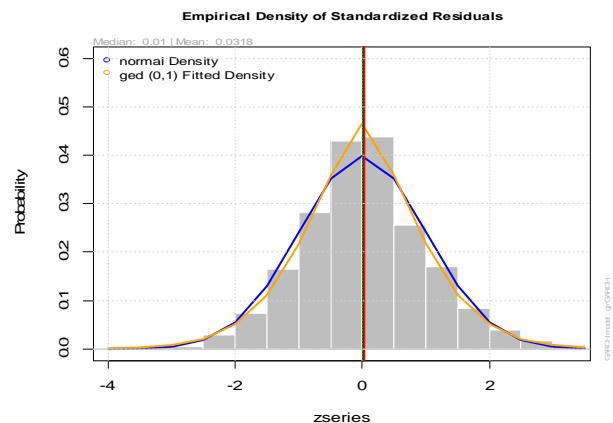


Figure 5. Density of Standardized Residuals for fitted TGARCH(1,1) model.

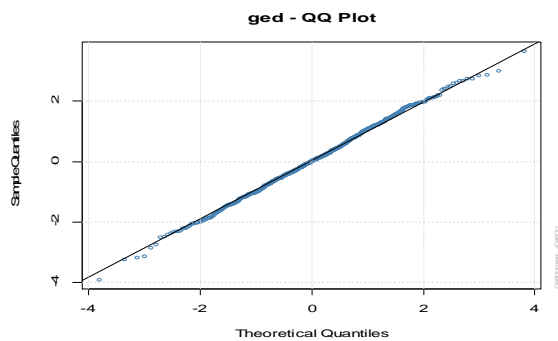


Figure 6. Normal QQ Plot for fitted TGARCH (1,1) model.

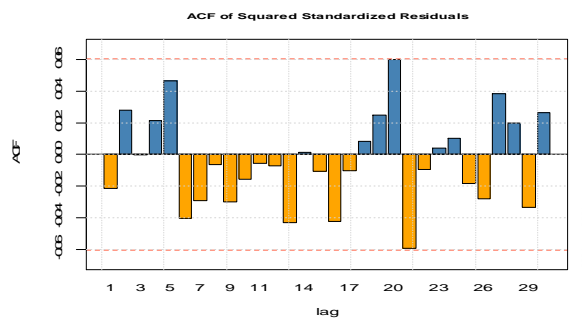


Figure 7. ACF of Squared Standardized Residuals for fitted TGARCH (1,1) model.

Figure 4 to Figure 7 represent that the fitted model is reasonably well. In other words, the model considered to best fit data will be the one with the distribution of standardized residuals the most close to normal distribution. Simultaneously, they should not have any autocorrelation. For proving asymmetry information Figure 8 represent the news impact curve. This is the strong indication that the TGARCH (1,1) model is appropriate.

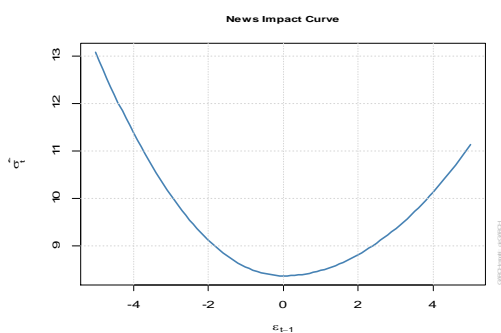


Figure 8. News impact curve for fitted TGARCH (1,1) model.

Once we have fitted the TGARCH which modeled the risk. We would like to check the model performance by performing historical backtest. For that, we can compare the estimated VaR (value at risk) with the actual return over the period. If the return is more negative than the VaR, we have a VaR exceedance. In our case, a VaR exceedance should only occur in 1% of the cases if we specified a 99% confidence level. VaR at 1% shows what 1% probability of my extreme loss.

VaR and Conditional VaR (CVaR) are the measures of risk measure in finance. Both measures are used to value at risk. CVaR generally considered as average potential losses. VaR and

CVaR are most commonly measure at three different ways, 95%, 99% and 99.9%. From Table 4.22 it is observed that VaR 95% is negative 1.53%, which means that there is a 5% chance, you lose 1.53% or more portfolio value in a single day. This value is derived from historical value so the values may differ.

Table 3. Value-at-Risk value of DS30 Return Index

VaR(5%)	-1.53%	CVaR(5%)	-2.13%
VaR(1%)	-2.72%	CVaR(1%)	-3.11%
VaR(0.1%)	-5.04%	CVaR(0.1%)	-4.83%

Similarly VaR 99% and VaR 99.9% are -2.72% and -5.04% respectively, which means that there is a 1% and 0.1% chance, you lose 2.72% and 5.04% respectively or more portfolio value in a single day. Finally exactly how much you loose on average on worst cases scenarios you have to look at CVaR values. CVaR 95% means, in the worst 5% returns the average loss will be 2.13%. Likewise, if CVaR 99% means, in the worst 1% returns the average loss will be 3.11% and for 99.9% the worst 0.1% returns the average loss will be 4.83%.

The outcome of our backtest with a rolling window is illustrated below in Figure 10. From Figure 10 it shows the number of exceedances for each VaR model, with an alpha level set at one percent for the backtest conducted between 2013 and 2017. The black line represents the Value-at-Risk level, forecasted for a period length of 925 observations with a one day moving window that refits every 120th step. All the returns are plotted and, as observed, some observations have returns lower than the Value-at-Risk level. These observations are called exceedances and are marked red in the graph.

The VaR accumulate 5 exceedances over the backtest period though the expected exceed was 9.2. Therefore, the TGARCH model may be the best estimating Value-at-Risk.

Table 4. Value-at-Risk Backtest Report of DS30 return index

Backtest Evaluation	Model: TGARCH-GED
Backtest Length	925
Alpha	1%
Expected Exceed	9.2
Actual VaR Exceed	5
Actual %:	0.5%
Unconditional Coverage (Kupiec)	
Null-Hypothesis	Correct Exceedances
LR.uc Statistic	2.368
LR.uc Critical	6.635
LR.uc p-value	0.124
Reject Null	NO

Table 4 provides statistics for our backtest evaluation results. Counting the number of exceedances, we find that the expected number of exceedances.

The test indicates that we cannot reject model because its exceedances are lower than the expected number. The results of the unconditional coverage test show that the null hypothesis holds for all models since neither of them have a likelihood ratio

statistic surpassing the critical value from the chi-square table. This is also indicated by the p-values, which are not below the 0.05 level associated with the 95 percent confidence level of the test. The confidence interval of the unconditional coverage test has a non-rejection region between 4 and 17. Thus, the null-hypothesis that the forecasted exceedances fall inside the correct interval cannot be rejected.

Figure 10 compare the number of expected versus actual exceedances given the tail probability of VaR, we can see the Kupiec's unconditional coverage test tests a test of the unconditional coverage of the exceedances. If the actual exceedances are smaller than the expected exceedances then we can't reject the null hypothesis that the exceedances are correct.

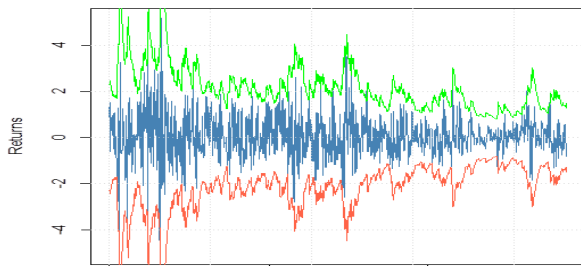


Figure 9. Series with 1% VaR limits

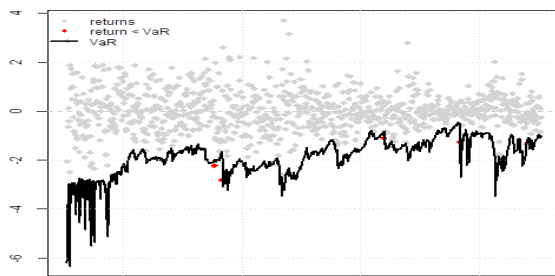


Figure 10. Daily returns of DS30 index and Value-at-Risk exceedances (VaR forecast) (alpha=0.01)

Here, in the Figure 9 the returns of the data (blue) hits the 1% VaR (Red) 5 times compare to 9.2 times expected. Hence we cannot state the rejection of the null hypothesis that the exceedances are correct. The p-values of Kupiec's unconditional coverage is not less than 5% and the Kupiec's unconditional coverage test suggests that the null hypothesis is not rejected. Hence model should be used in real life. So forecasting is very necessary however we will forecast for the next.

Table 5. Forecasted value of VaR

Time	Series	Sigma
T+1	-0.04665	0.5057
T+2	-0.01066	0.5124
T+3	0.01190	0.5191
T+4	0.02604	0.5257
T+5	0.03490	0.5321
T+6	0.04046	0.5385
T+7	0.04394	0.5449
T+8	0.04612	0.5511
T+9	0.04749	0.5573

T+10	0.04835	0.5634
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The forecasts for the volatility are given by sigma and 1% VaR is given just by the side for each period. For the period T+3, the expected volatility is 0.01190 and the 1% Value at Risk is 0.5191.

4. CONCLUSION

The conclusions arrived are the following:

To estimate the volatility is important to develop a model that contemplate the movements of the volatility within the time series, for that reason, the standard volatility don't follows the market variations, and also the forecast of the market power unit have in bound moment more than reserve and in alternative moments an absence of reserve, once we do the estimation exploitation this traditional volatility with a significant tail distribution.

The asymmetric GARCH models, like TGARCH model, solely fulfill with the movements of the volatility, as we will observe with the back testing conferred, conjointly it's necessary to use the significant tails distributions.

The time series history fulfills with the necessities of Basel Committee, to create the volatility forecast. It's simple to show this model to the traders benefit. The traders and also the investors solely the recent past import them.

The models as we are able to observe are dynamic, and is extremely vital the revision of those models periodically. The final impartial is extant a precise reserve that covers the utmost loss potential, and this reserves might have a value that corresponds with the truth. Once we announce the VaR restrictions for the traders and for the managers, the forecast of the reserve is also a reputable worth at impact that traders and managers use this information for decision-making. Conjointly the market VaR together with the credit risk and operational risk serves to calibrate the whole risk of a performance of the corporate.

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