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Evaluation of the Reliability of Fiber-Optic Information Transmission Systems Based on the Laws of Failure Distribution

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ABSTRACT- The influence of various failure distribution laws on the reliability of fiber-optic data transmission systems (FODTS) components is analyzed. The article discusses the failure distribution laws applicable to hardware and software components of the system, which allow taking into account their interaction and mutual influence on the overall reliability of the system. As a methodology, reliability assessment models are used, in particular, the exponential distribution, the Weibull distribution, and the Musa- Okumoto software reliability model. Formulas for assessing the overall reliability of the system and probability density functions are given, allowing a more complete picture of the stability and reliability of the FODTS under operating conditions.

KEYWORDS- Fiber-Optic Information Transmission Systems, Reliability, Failure, Failure Distribution Law, Software Reliability Model.

I. INTRODUCTION

The basis of modern telecommunications are FODTS, providing high-speed and reliable data transmission over long distances. Hardware and software reliability of FODTS are among the important factors for ensuring stable and trouble-free operation of telecommunication networks. Hardware reliability of the FODTS is defined as the ability of the hardware components of the system to perform their functions for a specified time under certain operating conditions, which depends on the quality of materials, design features, operating conditions, as well as the impact of external factors such as temperature, humidity, mechanical impacts, etc.[1][2][3]. Software reliability is associated with the ability of the software that controls the operation of the FODTS to function correctly and process data without errors for a specified time. Software reliability includes resistance to failures, protection from software errors and viruses, as well as the ability to self -recovery after failures [4][5][6]. This is ensured by high-quality code, effective algorithms, thorough testing and monitoring of software operation.

The conducted analysis shows that the study of distribution laws is one of the main factors in ensuring high reliability of hardware and software components of the FODTS [5], [7][8][9]. When analyzing the FODTS, it is usually assumed that the failure rate of all elements and components is the same or their reliability is the same, that is, all elements and components obey the same distribution law. However, in real conditions, the failure rate and reliability of various FODTS elements can vary significantly. The probability of failure-free operation of hardware and software is characterized by different distribution laws of a random variable. Therefore, one of the main tasks of increasing reliability is the study of complex distribution laws. In [7][8], complex distribution laws for two random variables with the same distribution laws, in particular exponential laws, are studied. This paper considers various distribution laws for hardware and software, which allow taking into account their specific features and increasing the overall reliability of the system.

II. RESEARCH METHOD

Random discrete or continuous variables $x_1, x_2, ..., x_n$ with a joint probability distribution $f(x_1, x_2, ..., x_n)$ and a marginal distribution, $f_1(x_1), f_2(x_2), ..., f_n(x_n)$ respectively, are mutually statistically independent if and only if [5]

$$f(x_1, x_2, ..., x_n) = f_1(x_1) \cdot f_2(x_2) \cdot ... \cdot f_n(x_n) \quad (1)$$

for all ($x_1, x_2, ..., x_n$) within their range.

Expression (1) is valid for discrete, continuous and mixed random variables [8]

In sequential systems, all components must be in a functioning state for the system to function, and its reliability is expressed as [7]:

$$P_{sys}(t) = \prod_{i=1}^{m} P_i(t) \quad (2)$$

where $P_i(t)$ is the reliability of the components.

Failure of hardware components of the FODTS, such as transceiver, amplifier, laser and software provision, for example, control systems, monitoring systems can obey different distribution laws. In order to find a generalized model of a system with independent components, a composition of distribution laws is performed [8].

In this paper, a generalized formula for the probability of failure-free operation for a FODTS consisting of a communication channel (optical fiber), an amplifier, a transceiver and software is compiled, each of which can obey different distribution laws.

The analysis showed that the transceiver failures [8][9][10] are most applicable to the exponential distribution for assessing the reliability of electronic parts such as semiconductor and laser diodes or photodiodes, where the "memory-free" properties simplify the calculation of failures after repair and a relatively accurate representation of the failure time of electronic components. After the initial "burnout" stage of components, failures occur randomly with a constant intensity, which is ideally modeled by an exponential law. Reliability function $P(t)$, which determines the probability of failure-free operation during time t [11]:

$$P(t)_{tr} = e^{-\lambda_{tr} t} \quad (4)$$

where λ_{tr} is the transceiver failure rate.

The uptime in this case is also distributed exponentially [12].

The fiber-optic communication channel and amplifier are subject to wear, aging and degradation, so the probability of failure may increase over time due to the influence of external factors and physical wear of the active components [11][12][13]. For these components, the Weibull distribution is most suitable, which allows taking into account the changing failure rate. The reliability function for the fiber-optic communication channel and amplifier is determined by the following expression [11][12][13]:

$$P(t)_{ch} = e^{-\left(\frac{t}{\beta_{ch}}\right)^{\alpha_{ch}}} \quad (5)$$

$$P(t)_{amp} = e^{-\left(\frac{t}{\beta_{amp}}\right)^{\alpha_{amp}}} \quad (6)$$

where α is the shape parameter, β is the scale parameter.

Software reliability is one of the important attributes that improves the quality of software. It is not easy to evaluate software reliability because there are a number of software reliability evaluation models that can be used to test and analyze failure data during software testing.

Based on the conducted analysis, a comparative table 1 of software models was compiled [14][15].

Table 1: Classification of Software Reliability Models

Classification	Subclass	Model	Description
Analytical models	Dynamic (continuous)	Jelinski-Moranda Model, Musa Model, Transition Probability Model, Musa-Okumoto Model	Model the growth of reliability over time, taking into account operational data.
	Dynamic (discrete)	Shooman Model, Modified Shooman Model, La Model Padula, Model Chic-Wolverton	Evaluate failure rates in real time
	Static models (by data domain)	Mills Model, Lipov Model, Simple Intuitive Model	Estimation of the distribution of errors in code and data.
	Static models (by error domain)	Corcoran Model , Nelson Model	Predicting data distribution for reliability
Empirical models	Complexity model	Analysis of software architecture and complexity of fixes	Predicting the Impact of Complexity on Software Reliability
	Model of program refinement time	Estimating time to achieve reliability	Predicting the time costs of software improvements
Software Development Life Cycle	Requirements development	Rome Laboratory Model, Musa Expectations Model	Early prediction of errors based on data collection
	Programming	Hybrid White Box Models, Hybrid Black Box Models	Testing models with or without access to code
	Testing	Jelinski-Moranda Model, Musa Model, Musa- Okumoto Model, Shooman Model	Forecasting the growth of software reliability

The Okumoto software model is the most applicable because it allows for the logarithmic nature of the failure rate reduction. This is especially important for software in complex systems, such as FODTS, where errors are corrected unevenly and early defects are fixed faster. The model also takes into account real failure data for reliability growth analysis, and its parameters can be easily adapted based on historical failure and defect elimination data [16].

In a state where the software is unstable, this indicates that its failure data follow a certain trend: reliability either increases or decreases, or first decreases and then increases. Using the Musa- Okumoto logarithmic Poisson runtime model and the model parameters obtained by the maximum likelihood method, an expression for the reliability function can be formulated [16]. The average value of the number of failures at time t is determined by the following formula [15]:

$$m(t) = \frac{1}{\theta}\ln(\lambda_0 \theta t + 1) \quad (7)$$

where $m(t)$ is the average value of the number of failures, λ_0 is the initial failure rate, θ is the rate of decrease in the failure rate at a given point in time.

Between the average value of the number of failures $m(t)$ and the failure rate $\lambda(t)$ have the following relationship [17][18]

$$m(t)=\int_0^t \lambda(t)dt. \quad (8)$$

The software reliability function can then be written:

$$P_{SW}(t)=e^{-\int_0^t \lambda(t)dt}=e^{-m(t)}=e^{-\frac{1}{\theta}\ln(\lambda_0\theta t+1)}. \quad (9)$$

III. RESULTS AND DISCUSSIONS

The overall reliability of the system based on formula (2) will be as follows:

$$P_{\text{sys}}(t)=P_{\text{tr}}(t)\cdot P_{\text{ch}}(t)\cdot P_{\text{amp}}(t)\cdot P_{\text{SW}}(t). \quad (10)$$

By substituting the reliability functions of the components and simplifying, we can obtain the following formula:

$$P_{\text{sys}}(t)=e^{-\left[\lambda_{\text{tr}}t+\left(\frac{t}{\beta_{\text{ch}}}\right)^{\alpha_{\text{ch}}}+\left(\frac{t}{\beta_{\text{amp}}}\right)^{\alpha_{\text{amp}}}+\frac{1}{\theta}\ln(\lambda_0\theta t+1)\right]}. \quad (11)$$

Time to failure distribution function [2,17] :

$$F(t)=1-P(t). \quad (12)$$

The probability density function of failures, which represents the probability that the system will fail at time *t* [2,17]:

$$f(t)=\frac{dF(t)}{dt}. \quad (13)$$

Based on formulas (11) in (12) and keeping in mind (13), we can obtain an expression for the probability density function:

$$f(t)=-\frac{dP_{\text{sys}}(t)}{dt}=e^{-\left[\lambda_{\text{tr}}t+\left(\frac{t}{\beta_{\text{ch}}}\right)^{\alpha_{\text{ch}}}+\left(\frac{t}{\beta_{\text{amp}}}\right)^{\alpha_{\text{amp}}}+\frac{1}{\theta}\ln(\lambda_0\theta t+1)\right]}\cdot \quad (14)$$
$$\cdot\left[\lambda_{\text{tr}}+\frac{\alpha_{\text{ch}}}{\beta_{\text{ch}}}\left(\frac{t}{\beta_{\text{ch}}}\right)^{\alpha_{\text{ch}}-1}+\frac{\alpha_{\text{amp}}}{\beta_{\text{amp}}}\left(\frac{t}{\beta_{\text{amp}}}\right)^{\alpha_{\text{amp}}-1}+\frac{\lambda_0}{\lambda_0\theta t+1}\right]$$

The correctness of the analytical models of reliability and probability density of failures is confirmed on the basis of the boundary conditions:

At $t=0$ system reliability $P(t)=1$, failure probability density $f(t)>0$ i.e.

$$P(t)\big|_{t=0}=1,\quad f(t)\big|_{t=0}>0, \quad (15)$$

with $t\to\infty$ system reliability

$$\lim_{t\to\infty}P(t)=0. \quad (16)$$

The given boundary conditions confirm the adequacy of analytical models (11), (14) to the actual operating conditions of the FODTS.

The graph, constructed based on the system reliability formula (Figure 1) using hypothetical data, shows a decrease in system reliability over time that corresponds to real operating conditions.

A comparative graph of the dependence of system reliability on the shape parameter and the scale parameter of the Weibull distribution, with fixed values of the exponential distribution parameters and the software reliability model, is shown in Fig. 2. It is evident from the graph (Figure 2) that an increase in the scale parameter β in the Weibull model leads to a slowdown in the decline in system reliability. This indicates that components with a higher scale parameter have a longer service life. The shape parameter α determines the rate of change in reliability, a high value of which indicates a sharp drop in system reliability.

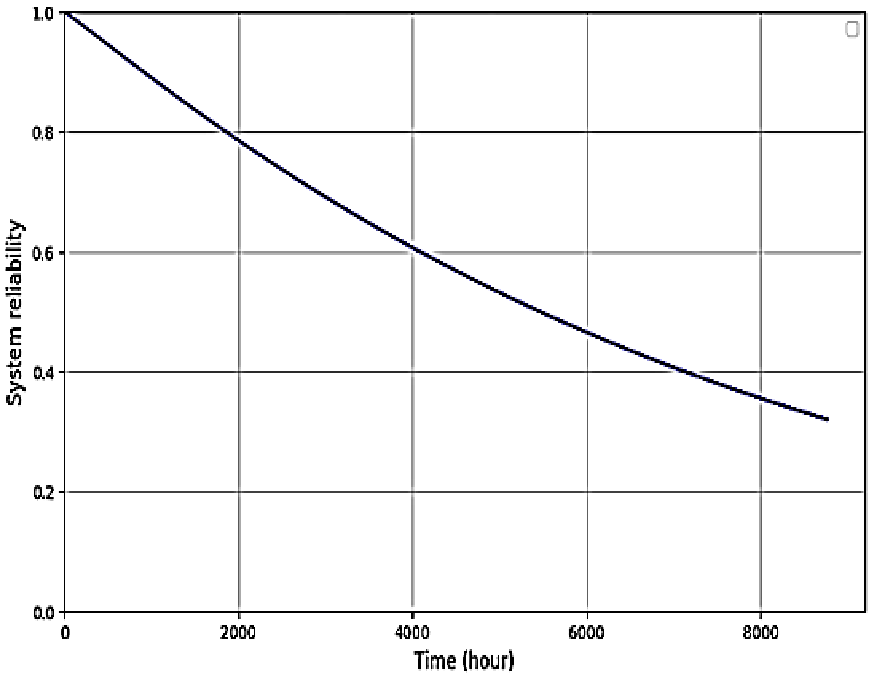

Figure 1: System Reliability Depending on Time

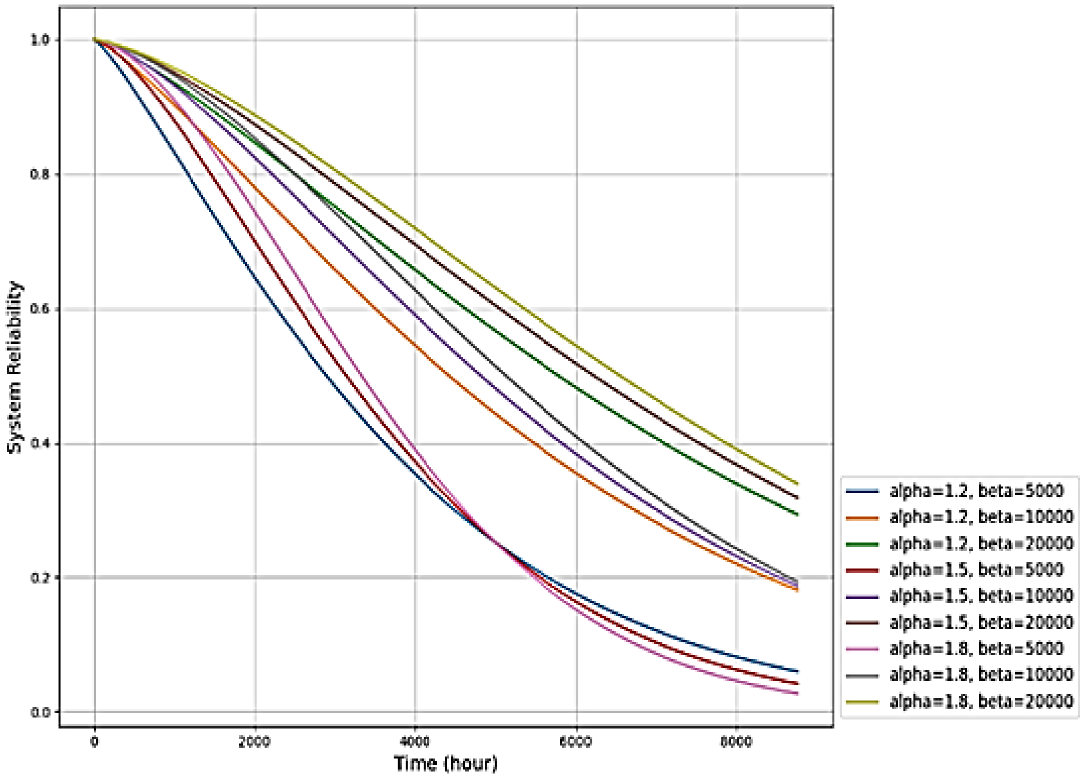

Figure 2: Reliability of the system for different values of the Weibull distribution parameters

The dependence of the system reliability on the parameters of the software reliability model at a constant value of the parameters of the Weibull and exponential distributions are shown in Figure 3 for the rate of decrease in the failure rate at time θ at a constant value of the failure rate and in Figure 4 for the failure rate λ_0 at a constant value of θ.

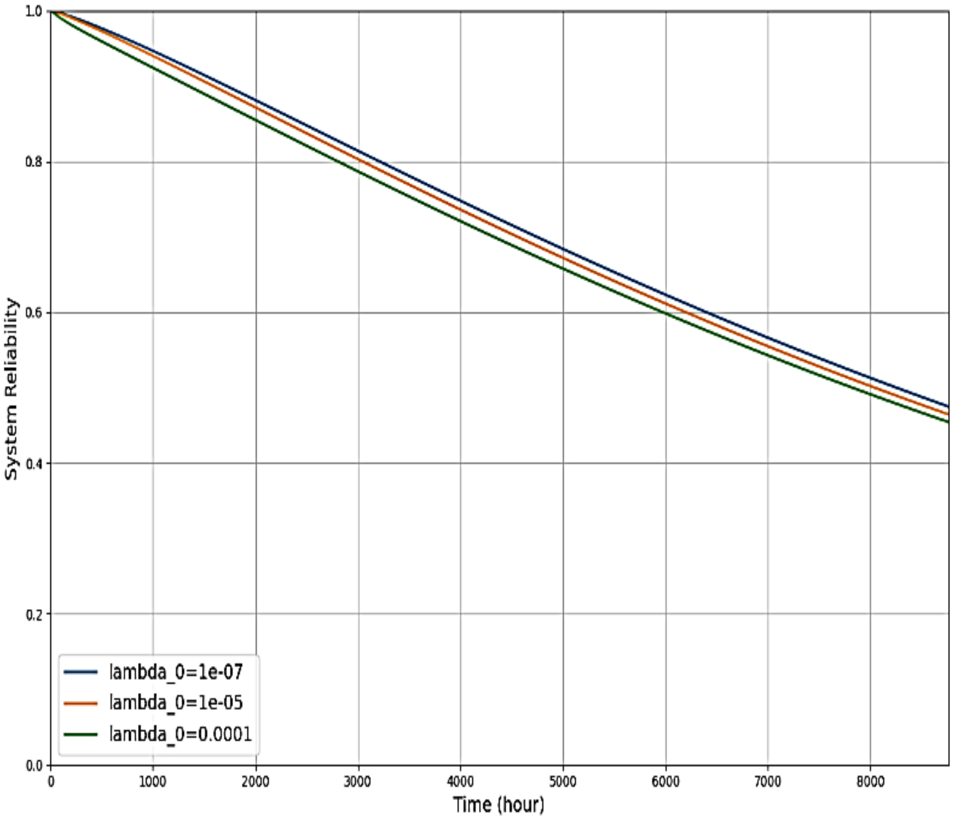

Figure 3: Dependence of system reliability on the parameter θ in the software model

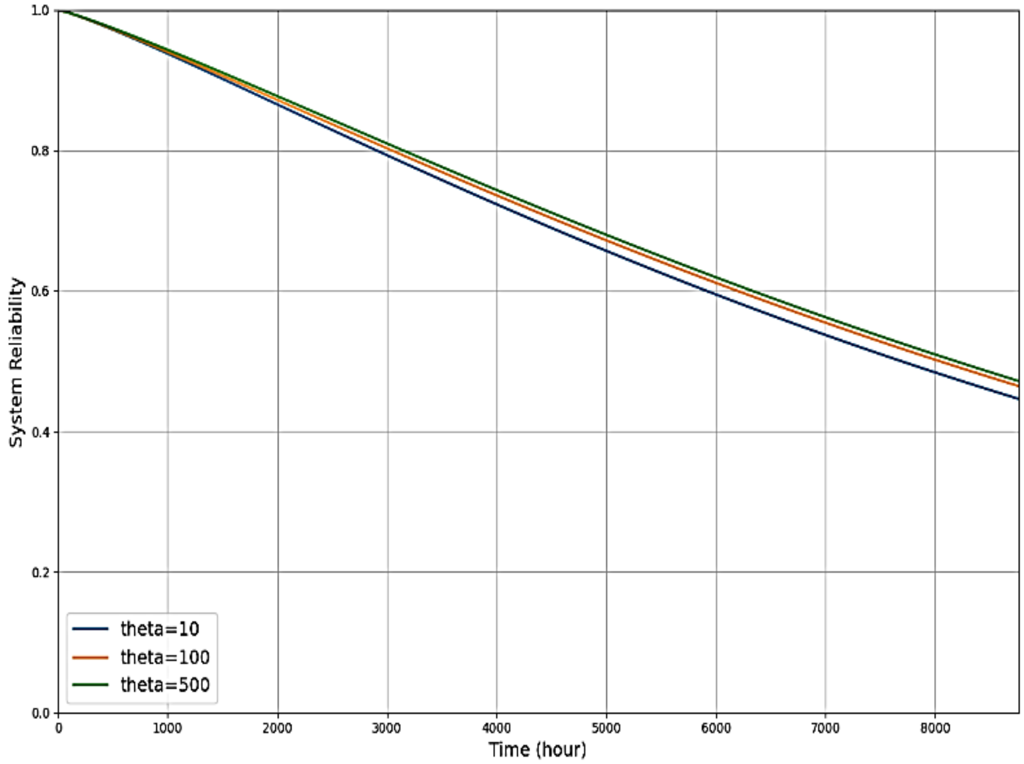

Figure 4: Effect of initial failure rate λ_0 on system reliability

The graphs of the dependence of system reliability on the parameters θ and λ_0 (Fig. 3, Fig. 4) of the Musa-Okumoto model show that with a high value of θ, the failure rate decreases faster and leads to a slower drop in system reliability, the increase of which is achieved by improving the processes of software development and testing. The initial failure rate significantly affects the reliability of the system, its lower values λ_0 demonstrate higher resistance to failures throughout the entire period of operation.

IV. CONCLUSION

The analysis showed that the choice of failure distribution models is important when assessing the reliability of the FODTS. The paper presents the most applicable distribution laws for the hardware and software components of the system. The boundary conditions and the presented graphs confirm the validity of analytical expressions in describing the behavior of the system reliability and the probability density of failures under FODTS operating conditions. To increase reliability, measures are needed to improve the quality and characteristics of each element of the system. A high value of the scale parameter depends on the durability and reliability of the components, the increase of which requires an increase in the quality of the components, improving their protection and the reliability of the design. To ensure high reliability of the system, it is important to reduce the value of the shape parameter, which is achieved through the use of high-quality materials, regular maintenance, the introduction of redundancy systems to reduce operational loads and monitoring of operating conditions also contributes to achieving the desired reliability characteristics. A decrease in the initial failure rate and an increase in the rate of decrease in the failure rate are achieved by improving the processes of software development, testing and operation. The implementation of these measures will extend the service life of the system, reduce the failure rate and increase the reliability of the FODTS.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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